

Development of GPS-Based Attitude Determination Algorithms

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This paper describes two Global Positioning System (GPS) based attitude determination algorithms which contain steps of integer ambiguity resolution and attitude computation. The first algorithm extends the ambiguity function method to account for the unique requirement of attitude determination. The second algorithm explores the artificial neural network approach to find the attitude. A test platform is set up for verifying these algorithms.

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I. INTRODUCTION

Attitude determination is a problem that is central to the navigation system design of spacecraft and aircraft. Previously, this problem is accounted for using specialized sensors such as inertial sensors, sun sensors, star trackers, etc., together with sophisticated algorithms to facilitate the attitude solution; see [1, 2] for details. The determination of spacecraft and aircraft attitude using the Global Positioning System (GPS) measurements has many advantages over existing attitude determination methods. But there exist several problems that should be addressed including real-time requirement, low upgrade rate, low accuracy with selective availability and multipath. We develop two algorithms for attitude determination by measuring of phase difference of the GPS carrier signal observed at two antennas. Previously there have been several papers that addressed the same problem using the triple difference of the GPS carrier phase measurements [3–5], to alleviate the hard problem of ambiguity resolution. But the triple difference is subject to significant noise contamination. In addition, once there is a cycle slip in one receiver due to low signal to noise ratio or loss of track of the Doppler filter, the error will propagate and obscure the attitude determination algorithm. So the double difference measurements are used to furnish this attitude determination in [7–10]. In order to determine the attitude from GPS carrier phase measurements, we have to resolve ambiguities [11, 12]. In other words, we have to find the correct carrier phase integer ambiguity values. This is the key to GPS attitude determination. Of course, there are other methods to measure the carrier phase using single difference interferometric measurements [13]. In this work, an equation that governs the relationship among the attitude, integer ambiguity, and double difference measurement is developed. Two algorithms are then proposed to determine the attitude (azimuth) and resolve the integer ambiguity in real time. The first algorithm utilizes the concept of ambiguity function method to formulate the problem as an optimization problem on the attitude. The attitude is then solved using a line search method. The second method computes candidate attitude solutions. Based upon the tabulated attitude candidates, the correct combination of integers is then resolved using a min-max type optimization method. A neural network architecture is proposed to realize the min-max optimization. The algorithms have been verified by setting up a platform and taking appropriate measurements.

II. SYSTEM DYNAMICS AND MEASUREMENTS

The attitude can be characterized using a direction cosine matrix, Euler angles, quaternions elements,

and Cartesian position vectors [1]. It governs the relationship between two coordinate systems. In this section, the attitude representation of an Earth-centered Earth-fixed platform is discussed. The GPS carrier phase measurements are then related to the attitude representation.

A. Platform Dynamic Model

The 3×3 attitude matrix C_e^b , which transforms vectors from the Earth-centered Earth-fixed e -frame to the body-fixed b -frame, can be regarded as a sequence of transformations from the e -frame to an intermediate frame and then from the intermediate frame to the b -frame. In terms of matrix rotation, we have

$$C_e^b(t) = C_m^b(t)C_e^m(t)$$

where the m -frame is an intermediate local level frame which has its first-axis points to the east, second-axis to the north, and third-axis up. Consequently, the direction cosine matrix $C_e^m(t)$ that governs the transformation from the Earth-centered Earth-fixed e -frame to the local level m -frame is characterized by the latitude and longitude according to

$$C_e^m(t) = R_1\left(\frac{\pi}{2} - \phi(t)\right)R_3\left(\frac{\pi}{2} + \lambda(t)\right)$$

where $\phi(t)$ and $\lambda(t)$ are, respectively, the latitude and longitude at the test site. The notation R_i denotes a rotation about the i th axis. For any angle ν ,

$$R_1(\nu) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \nu & \sin \nu \\ 0 & -\sin \nu & \cos \nu \end{bmatrix},$$

$$R_2(\nu) = \begin{bmatrix} \cos \nu & 0 & -\sin \nu \\ 0 & 1 & 0 \\ \sin \nu & 0 & \cos \nu \end{bmatrix},$$

$$R_3(\nu) = \begin{bmatrix} \cos \nu & \sin \nu & 0 \\ -\sin \nu & \cos \nu & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Note also that for a vector r , its representation in the e -frame, denoted by $r^{[e]}$, is related to that in the m -frame, $r^{[m]}$, by

$$r^{[m]} = C_e^m(r^{[e]} - r_s^{[e]})$$

where r_s is the vector from the geocenter to the origin of the m -frame. Assume that the platform is mounted with its vertical axis pointing up. The matrix $C_m^b(t)$ can then be represented using a direction cosine matrix

$$C_m^b(t) = R_3(\psi(t))$$

where $\psi(t)$ is the angle between the principal axis of the platform and the east direction. We can also call this angle the azimuth of the platform.

B. GPS Carrier Phase Measurement

The carrier phase measurement at site m with respect to the satellite i admits the relationship [14]

$$\Phi_m^i(t) = \frac{f}{c} \varrho_m^i(t) + N_m^i + f[\delta_m(t) - \delta^i(t)] - \frac{\kappa^i}{cf} \quad (1)$$

where $\Phi_m^i(t)$ is the carrier phase measurement, f is the carrier frequency, c is the speed of light, $\varrho_m^i(t)$ is the distance between the observation site m and the satellite i , $\delta_m(t)$ is the receiver clock error, δ^i is the satellite clock error, N_m^i is the associated integer ambiguity, and κ^i which is related to the total electron content along the GPS signal transmission path stands for the ionospheric effect and the tropospheric error.

The topocentric distance $\varrho_m^i(t)$ from the m th antenna to the i th GPS satellite is known to be

$$\varrho_m^i(t) = \|C_b^e a_m^{[b]} - (s_i^{[e]} - r_s^{[e]})\| \quad (2)$$

where C_b^e is the direction cosine matrix from the body-fixed b -frame to the Earth-centered Earth-fixed coordinate system. $a_m^{[b]}$ is the position (a 3×1 vector) of the m th antenna in the body-fixed frame, $s_i^{[e]}$ is the position vector, offset by r_s of the i th GPS satellite in the e -frame.

Substituting (2) into (1) the carrier phase measurement equation yields

$$\begin{aligned} \Phi_m^i(t) &= \frac{f}{c} \|C_b^e a_m^{[b]} - s_i^{[e]}\| + N_m^i + f[\delta_m(t) - \delta^i(t)] - \frac{\kappa^i}{cf} \\ &= \frac{f}{c} \|C_b^e a_m^{[b]} - s_i^{[e]}\| + N_m^i + e_m + g_i \end{aligned} \quad (3)$$

where e_m and g_i , which are clear from the equation, represent the resultant errors associated with the m th antenna and the i th satellite, respectively.

C. Single, Double, and Triple Differences

The single difference between the carrier phase measurements at the two sites m and n with respect to the satellite i is obtained by taking the difference between two carrier phase measurements:

$$\Phi_{mn}^i(t) = \Phi_n^i(t) - \Phi_m^i(t) = \frac{f}{c} \varrho_{mn}^i(t) + N_{mn}^i + e_{mn}(t) \quad (4)$$

where

$$\varrho_{mn}^i(t) = \varrho_n^i(t) - \varrho_m^i(t),$$

$$N_{mn}^i = N_n^i - N_m^i, \quad \text{and}$$

$$e_{mn}(t) = e_n(t) - e_m(t).$$

Some common errors associated with the i th satellite, in particular, the satellite clock error, selective availability, ionospheric effect, and tropospheric error, are canceled out by taking the single difference.

Note that the topocentric distance between the m th antenna and i th satellite can be approximated as

$$\begin{aligned} \varrho_m^i &= \left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} + C_b^e \left(\frac{a_m^{[b]} - a_n^{[b]}}{2} \right) \right\| \\ &\approx \left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\| \\ &\quad + \left(\frac{a_m^{[b]} - a_n^{[b]}}{2} \right)^T C_b^e \frac{C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]}}{\left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\|}. \end{aligned} \quad (5)$$

This approximation is valid since the baseline length between the two antennas is much smaller than the topocentric distance. Similarly,

$$\begin{aligned} \varrho_n^i &\approx \left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\| \\ &\quad + \left(\frac{a_n^{[b]} - a_m^{[b]}}{2} \right)^T C_b^e \frac{C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]}}{\left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\|}. \end{aligned}$$

Hence, the difference between $\varrho_m^i(t)$ and $\varrho_n^i(t)$ becomes

$$\varrho_{mn}^i = (a_n^{[b]} - a_m^{[b]})^T C_b^e \frac{C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]}}{\left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\|}. \quad (6)$$

Substituting (6) into (4) yields

$$\begin{aligned} \Phi_{mn}^i(t) &= \frac{f}{c} (a_n^{[b]} - a_m^{[b]})^T C_b^e \frac{C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]}}{\left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\|} \\ &\quad + N_{mn}^i + e_{mn}(t). \end{aligned} \quad (7)$$

The double difference involves two observation sites and two satellites. Let $\Phi_{mn}^i(t)$ and $\Phi_{mn}^j(t)$ be, respectively, the single differences with respect to satellites i and j , that is,

$$\begin{aligned} \Phi_{mn}^i(t) &= \frac{f}{c} \varrho_{mn}^i(t) + N_{mn}^i + e_{mn}(t) \\ \Phi_{mn}^j(t) &= \frac{f}{c} \varrho_{mn}^j(t) + N_{mn}^j + e_{mn}(t). \end{aligned}$$

The double difference is obtained by taking the difference between the two single differences. This leads to

$$\Phi_{mn}^{ij} = \Phi_{mn}^j(t) - \Phi_{mn}^i(t) = \frac{f}{c} \varrho_{mn}^{ij}(t) + N_{mn}^{ij} \quad (8)$$

where

$$\begin{aligned} \varrho_{mn}^{ij}(t) &= \varrho_{mn}^j(t) - \varrho_{mn}^i(t) \\ &= \varrho_n^j(t) - \varrho_n^i(t) - \varrho_m^j(t) + \varrho_m^i(t) \\ N_{mn}^{ij} &= N_{mn}^j - N_{mn}^i = N_n^j - N_n^i - N_m^j + N_m^i. \end{aligned}$$

The receiver clock biases are canceled by using the double difference.

Substituting (7) into (8), we obtain

$$\begin{aligned} \Phi_{mn}^{ij} &= \frac{f}{c} (a_n^{[b]} - a_m^{[b]})^T C_b^e \\ &\quad \times \left[\frac{C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_j^{[e]}}{\left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_j^{[e]} \right\|} - \frac{C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]}}{\left\| C_b^e \left(\frac{a_m^{[b]} + a_n^{[b]}}{2} \right) - s_i^{[e]} \right\|} \right] \\ &\quad + N_{mn}^{ij}. \end{aligned} \quad (9)$$

Note further that $a_m^{[b]} + a_n^{[b]}/2$ is much smaller than $s_j^{[e]}$, and therefore the double difference equation (9) can be approximated as

$$\Phi_{mn}^{ij} = \frac{f}{c} (a_n^{[b]} - a_m^{[b]})^T C_b^e \left[\frac{s_i^{[e]}}{\|s_i^{[e]}\|} - \frac{s_j^{[e]}}{\|s_j^{[e]}\|} \right] + N_{mn}^{ij}. \quad (10)$$

Suppose that there are n_a GPS receivers and n_s GPS satellites being simultaneously observed, there will be $(n_a - 1) \times (n_s - 1)$ linearly independent double differences. In the followings, methods for resolving the attitude matrix C_e^b and integer ambiguities N_{mn}^{ij} from (10) are presented. The double difference equation (10) can indeed be recast as a matrix equation

$$\Phi = AC_e^b S + N \quad (11)$$

where

$$\begin{aligned} \Phi &= \begin{bmatrix} \Phi_{12}^{12} & \Phi_{12}^{13} & \cdots & \Phi_{12}^{1n_s} \\ \Phi_{13}^{12} & \Phi_{13}^{13} & \cdots & \Phi_{13}^{1n_s} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{1n_a}^{12} & \Phi_{1n_a}^{13} & \cdots & \Phi_{1n_a}^{1n_s} \end{bmatrix} \\ A &= \frac{f}{c} \begin{bmatrix} (a_2^{[b]} - a_1^{[b]})^T \\ (a_3^{[b]} - a_1^{[b]})^T \\ \vdots \\ (a_{n_a}^{[b]} - a_1^{[b]})^T \end{bmatrix} \\ S &= \begin{bmatrix} \frac{s_1^{[e]}}{\|s_1^{[e]}\|} - \frac{s_2^{[e]}}{\|s_2^{[e]}\|} & \frac{s_1^{[e]}}{\|s_1^{[e]}\|} - \frac{s_3^{[e]}}{\|s_3^{[e]}\|} & \cdots & \frac{s_1^{[e]}}{\|s_1^{[e]}\|} - \frac{s_{n_s}^{[e]}}{\|s_{n_s}^{[e]}\|} \end{bmatrix} \\ N &= \begin{bmatrix} N_{12}^{12} & N_{12}^{13} & \cdots & N_{12}^{1n_s} \\ N_{13}^{12} & N_{13}^{13} & \cdots & N_{13}^{1n_s} \\ \vdots & \vdots & \ddots & \vdots \\ N_{1n_a}^{12} & N_{1n_a}^{13} & \cdots & N_{1n_a}^{1n_s} \end{bmatrix}. \end{aligned}$$

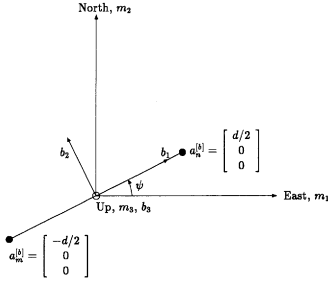


Fig. 1. Relationship between m -frame and b -frame.

One important observation concerning this formulation is that the measurement is linear in terms of the unknowns N and C_e^b .

III. ATTITUDE DETERMINATION

The direction cosine matrix $C_m^b(t)$ can be expressed as

$$C_m^b(t) = R_3(\psi(t))$$

where $\psi(t)$ is the angle between the principal axis of the platform and the east direction. The direction cosine matrix C_e^b can be expanded as

$$\begin{aligned} C_e^b(t) &= C_m^b(t)C_e^m(t) = R_3(\psi(t))R_1\left(\frac{\pi}{2} - \phi(t)\right)R_3\left(\frac{\pi}{2} + \lambda(t)\right) \\ &= \begin{bmatrix} -\cos\psi\sin\lambda - \sin\psi\sin\phi\cos\lambda & \cos\psi\cos\lambda - \sin\psi\sin\phi\sin\lambda & \sin\psi\cos\phi \\ \sin\psi\sin\lambda - \cos\psi\sin\phi\cos\lambda & -\sin\psi\cos\lambda - \cos\psi\sin\phi\sin\lambda & \cos\psi\cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}. \end{aligned}$$

Assume that there are two antennas located at

$$a_m^{[b]} = \begin{bmatrix} -d/2 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad a_m^{[e]} = \begin{bmatrix} d/2 \\ 0 \\ 0 \end{bmatrix},$$

respectively.

The variable d is the baseline length between the two antennas. The relationship between the local level m -frame and the body-fixed b -frame is shown in Fig. 1. The double difference equation (10) can then be simplified as

$$\begin{aligned} \Phi_{mn}^{ij} &= \frac{df}{c}[-\cos\psi\sin\lambda - \sin\psi\sin\phi\cos\lambda \\ &\quad \cos\psi\cos\lambda - \sin\psi\sin\phi\sin\lambda \quad \sin\psi\cos\phi] \\ &\quad \times \left(\frac{s_i^{[e]}}{\|s_i^{[e]}\|} - \frac{s_j^{[e]}}{\|s_j^{[e]}\|} \right) + N_{mn}^{ij}. \end{aligned} \quad (12)$$

In the above, the variables λ and ϕ are obtained from the GPS positioning data, $s_i^{[e]}$ and $s_j^{[e]}$ are obtained from the satellite ephemeris data, and Φ_{mn}^{ij} is the double difference of the GPS carrier phase measurements. The unknowns are the azimuth angle ψ and the integer ambiguity N_{mn}^{ij} .

Let

$$P'(t_k) = \frac{df}{c}[-\sin\lambda \quad \cos\lambda \quad 0] \left(\frac{s_i^{[e]}}{\|s_i^{[e]}\|} - \frac{s_j^{[e]}}{\|s_j^{[e]}\|} \right)$$

and

$$\begin{aligned} Q'(t_k) &= \frac{df}{c}[-\sin\phi\cos\lambda \quad -\sin\phi\sin\lambda \quad \cos\phi] \\ &\quad \times \left(\frac{s_i^{[e]}}{\|s_i^{[e]}\|} - \frac{s_j^{[e]}}{\|s_j^{[e]}\|} \right). \end{aligned}$$

Then, (12) can be rearranged as

$$\Phi_{mn}^{ij}(t_k) = P'(t_k)\cos\psi + Q'(t_k)\sin\psi + N_{mn}^{ij} \quad (13)$$

where t_k in the parenthesis is used to denote different time instant. Assume that the platform is fixed with respect to the e -frame and that ψ is not a function of t . Furthermore, assume that there are no cycle slips, so the integer N_{mn}^{ij} is a constant.

Based upon the analytic relationship between the attitude, integer ambiguity, and GPS measurement, two algorithms are developed in the following. The

first algorithm explores the integer constraint on the integer ambiguity and formulates the problem as an optimization problem on only one unknown: the attitude. The attitude is then solved through a line search. The second algorithm, on the other hand, resolves the candidate attitudes first and performs a search to determine the correct integer combination. This leads to a min-max type problem. A neural network is then proposed to solve the min-max optimization problem.

A. An Optimization Approach—Ambiguity Function Method

One method to obtain the azimuth angle ψ based upon a set of measurements is the ambiguity function method. Define the objective function J as

$$\begin{aligned} J &= \sum_{t_k} \sum_{ij} |e^{j2\pi(\Phi_{mn}^{ij}(t_k) - P'(t_k)\cos\psi - Q'(t_k)\sin\psi)} - 1|^2 \\ &\quad + \sum_{t_k} \sum_{ij} |\Phi_{mn}^{ij}(t_k) - \Phi_{mn}^{ij}(t_{k-1}) - [P'(t_k) - P'(t_{k-1})] \\ &\quad \quad \times \cos\psi - [Q'(t_k) - Q'(t_{k-1})]\sin\psi|^2. \end{aligned}$$

The first term on the right-hand side penalizes the deviation of N_{mn}^{ij} from an integer. The second term, on the other hand, ensures that the integer ambiguities are consistent. Note that the inclusion of the first term resembles the ambiguity function approach, which has been extensively investigated in resolving the integer ambiguity for position fixing. The second term, however, has been previously ignored in GPS-based precise positioning or attitude determination approaches. Note that even when the first term is minimized, it may happen, even in the absence of cycle slip, that there are jumps of integer between different time instants, leading to erroneous results. By incorporating both terms into the optimization setup, the desired angle can be accurately determined. Applying Euler's formula, the objective function can be rewritten as

$$J = \sum_{t_k} \sum_{ij} [2 - 2\cos(2\pi(\Phi_{mn}^{ij}(t_k) - P'(t_k)\cos\psi - Q'(t_k)\sin\psi))] + \sum_{t_k} \sum_{ij} |\Delta\Phi_{mn}^{ij}(t_k) - \Delta P'(t_k)\cos\psi - \Delta Q'(t_k)\sin\psi|^2$$

where

$$\Delta\Phi_{mn}^{ij}(t_k) = \Phi_{mn}^{ij}(t_k) - \Phi_{mn}^{ij}(t_{k-1})$$

$$\Delta P'(t_k) = P'(t_k) - P'(t_{k-1})$$

$$\Delta Q'(t_k) = Q'(t_k) - Q'(t_{k-1}).$$

The objective function depends only on the azimuth angle ψ , which can be solved using several search methods. Indeed, since ψ ranges between 0 and 2π , one can evaluate the function J for different ψ s and determine the optimal ψ .

B. Competitive Hopfield Neural Network Approach

The GPS-based attitude determination problem can be formulated as a min-max type optimization problem. Indeed, from the double difference equation (13), we have

$$\sqrt{(P'(t_k))^2 + (Q'(t_k))^2} [\sin\theta\cos\phi + \cos\theta\sin\phi] = \Phi_{mn}^{ij}(t_k) - N_{mn}^{ij} \quad (14)$$

where the angle θ is resolved from $P'(t_k)$ and $Q'(t_k)$ by

$$\theta = \tan^{-1} \frac{P'(t_k)}{Q'(t_k)}.$$

Thus, for any fixed integer N_{mn}^{ij} , there are two angles ψ s that satisfy the double difference equation, which can be solved from (14). Let

$$\alpha = \sin^{-1} \frac{\Phi_{mn}^{ij}(t_k) - N_{mn}^{ij}}{\sqrt{(P'(t_k))^2 + (Q'(t_k))^2}}. \quad (15)$$

Then, the solutions are, respectively,

$$\psi = \alpha - \theta \bmod 2\pi \quad \text{and} \quad \pi - \alpha - \theta \bmod 2\pi. \quad (16)$$

In order for α to exist, we must have

$$-\sqrt{(P'(t_k))^2 + (Q'(t_k))^2} \leq \Phi_{mn}^{ij}(t_k) - N_{mn}^{ij} \leq \sqrt{(P'(t_k))^2 + (Q'(t_k))^2}. \quad (17)$$

Equation (17) then governs the range in which the integer N_{mn}^{ij} can lie. Note that

$$(P'(t_k))^2 + (Q'(t_k))^2 = \left(\frac{df}{c}\right)^2 \left(\frac{s_i^{[el]}}{\|s_i^{[el]}\|} - \frac{s_j^{[el]}}{\|s_j^{[el]}\|}\right)^T \times \left\{ \begin{bmatrix} -\sin\lambda \\ \cos\lambda \\ 0 \end{bmatrix} [-\sin\lambda \quad \cos\lambda \quad 0] + \begin{bmatrix} -\sin\phi\cos\lambda \\ -\sin\phi\sin\lambda \\ \cos\phi \end{bmatrix} \right\} \times [-\sin\phi\cos\lambda \quad -\sin\phi\sin\lambda \quad \cos\phi] \times \left(\frac{s_i^{[el]}}{\|s_i^{[el]}\|} - \frac{s_j^{[el]}}{\|s_j^{[el]}\|}\right).$$

The matrix in the bracelet is less than or equal to the identity matrix (in the sense of matrix definiteness), thus,

$$(P'(t_k))^2 + (Q'(t_k))^2 \leq \left(\frac{df}{c}\right)^2 \left\| \frac{s_i^{[el]}}{\|s_i^{[el]}\|} - \frac{s_j^{[el]}}{\|s_j^{[el]}\|} \right\|^2 \leq 4 \left(\frac{df}{c}\right)^2.$$

This gives the bounds on the integer ambiguity N_{mn}^{ij} .

For each baseline combination mn , suppose that there are K possible integer values N_{mn}^{ij} . A matrix H of dimension $K \times (n_s - 1)$ can be constructed where the (k, l) th entry represents the angle obtained with respect to the l th (or more precisely, $1(l+1)$) double difference measurement while assuming that the integer is the k th index. The matrix has the following form

$$H = \{h_{kl}\} = \begin{bmatrix} (\psi_{1a}^{12}, \psi_{1b}^{12}) & (\psi_{1a}^{13}, \psi_{1b}^{13}) & \cdots & (\psi_{1a}^{1n_s}, \psi_{1b}^{1n_s}) \\ (\psi_{2a}^{12}, \psi_{2b}^{12}) & (\psi_{2a}^{13}, \psi_{2b}^{13}) & \cdots & (\psi_{2a}^{1n_s}, \psi_{2b}^{1n_s}) \\ \vdots & \vdots & \ddots & \vdots \\ (\psi_{Ka}^{12}, \psi_{Kb}^{12}) & (\psi_{Ka}^{13}, \psi_{Kb}^{13}) & \cdots & (\psi_{Ka}^{1n_s}, \psi_{Kb}^{1n_s}) \end{bmatrix}.$$

The problem is then reduced to finding an attitude angle ψ which appears in each column of H at least

once. The corresponding row in which the angle appears is thus the resulting integer ambiguity. Although each entry in the matrix H contains two angles. The problem can be tackled easily by augmentation technique, i.e., by splitting the matrix into a larger dimension $2K \times (n_s - 1)$ one. The attitude determination problem can be cast as a min-max type optimization problem

$$\min_{\psi} \left(\max_l \left(\min_k |h_{kl} - \psi| \right) \right). \quad (18)$$

This min-max optimization problem can then be solved using the neural network approach [15]. Here, a modified Hopfield network, called the competitive Hopfield neural network (CHNN) [16], is proposed for determining the attitude. The CHNN network differs from the original Hopfield network in that a winner-take-all rule is adopted to guarantee a feasible result. Consequently, the limitation of local convergence of the original Hopfield network is alleviated. In the CHNN, a competitive winner-take-all rule is imposed for the updating of neurons. The neurons in each column compete with one another in the same column to determine the neuron which receives the minimum input. The winner neuron sets its output to be one, and all the other neurons in this column are set to zero.

The procedure of finding the azimuth angle ψ using the competitive Hopfield neural network algorithm is described in the following. Instead of searching for the angle ψ directly, the neural network attempts to construct a permutation matrix $V = \{v_{kl}\}$ of the same size as H that bears the membership information of each entry in H . When $v_{kl} = 1$, then the l th double difference measurement has an integer ambiguity as the k th entry. Otherwise $v_{kl} = 0$. Since there is only one integer ambiguity with respect to each double difference measurement, each column of V has one and only one entry that takes the value one. In terms of the terminology in CHNN, the matrix becomes a pattern to be searched so that a certain energy is minimized with respect to the double difference measurements. The energy function must reflect the requirement in (18) and is selected to be

$$E = \frac{1}{2} \sum_r \sum_s \sum_k \sum_l |h_{rs} - h_{kl}| \cdot v_{rs} \cdot v_{kl}.$$

In other words, the difference $|h_{rs} - h_{kl}|$ becomes the weighting in the energy function to govern the optimization process. Once an initial configuration V is set up, the CHNN network begins to evolve based upon the double difference measurement until a new configuration is obtained such that the energy is minimized. This training or iteration process evaluates the network parameter

$$\text{net}_{rs} = - \sum_k \sum_l |h_{rs} - h_{kl}| \cdot v_{kl}$$

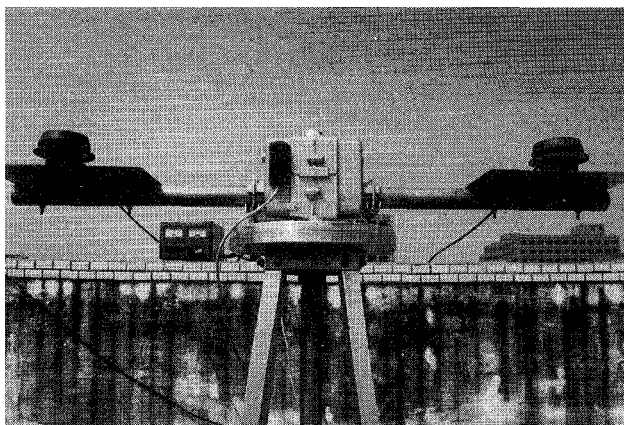


Fig. 2. Attitude determination system.

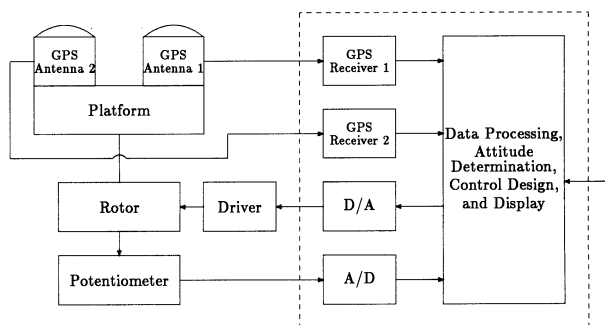


Fig. 3. System functional block diagram.

and assigns the new configuration as

$$v_{rs} = \begin{cases} 1 & \text{if } \text{net}_{rs} = \max_r(\text{net}_{rs}) \\ 0 & \text{otherwise} \end{cases}.$$

The process repeats until the energy function is reduced to its minimal level or the so-called stable state. The CHNN is found to always converge to a stable state in the network evolutions. Therefore, the process is guaranteed to be convergent. The procedure can be implemented on-line. Also, once the direction cosine matrix C_m^b is obtained, the attitude can be resolved by standard procedures.

IV. EXPERIMENTAL RESULTS

An experiment is set up to assess the two proposed attitude determination algorithms. Two GPS antennas are mounted at the ends of a 1 m long baseline as shown in Fig. 2. A schematic diagram of the experiment is shown in Fig. 3. A two-degree-of-freedom rotor is used to emulate the spin-stabilized satellite. The speed of rotation and the tilt angle can be changed by adjusting the rotor control voltage. The orientation of the platform is measured through a set of potentiometers mounted in the rotor. The potentiometer output serves as a reference angle of the platform. The potentiometers are calibrated each time before gathering data. The

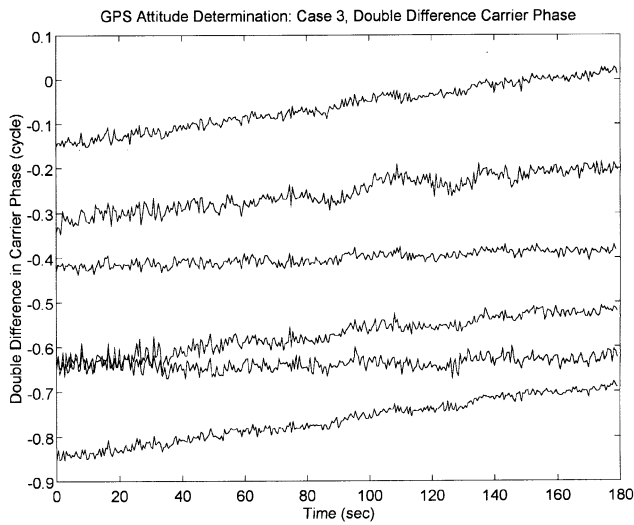


Fig. 4. Carrier phase double difference.

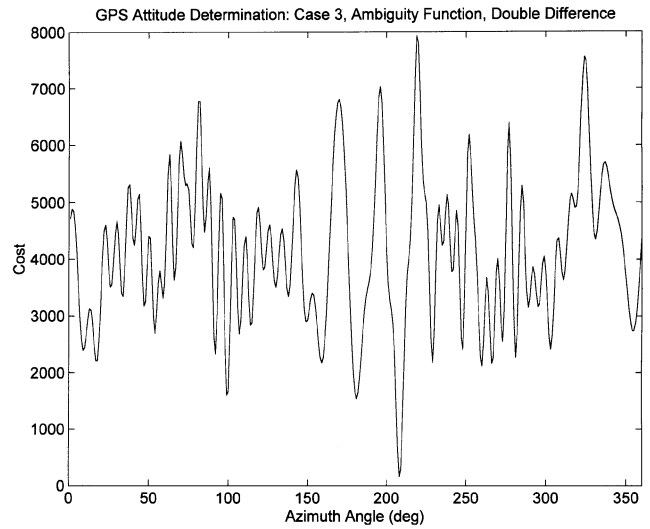


Fig. 5. Attitude determination by ambiguity function method.

TABLE I
Candidate Attitude Solutions Matrix

$$H = \begin{pmatrix} (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ (\infty, \infty) & (\infty, \infty) & (125.9378 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & & 177.4140) & & & \\ (\infty, \infty) & (175.0996 & (114.1475 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & -152.1153) & -170.7957) & & & \\ (\infty, \infty) & (157.4851 & (104.9363 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & -134.5008) & -161.5844) & & & \\ (\infty, \infty) & (145.8059 & (96.9563 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & -122.8215) & -153.6044) & & & \\ (\infty, \infty) & (136.1173 & (89.6999 & (169.8627 & (\infty, \infty) & (\infty, \infty) \\ & -113.1330) & -146.3480) & -153.8515) & & \\ (\infty, \infty) & (127.4572 & (82.9054 & (147.0233 & (\infty, \infty) & (119.9537 \\ & -104.4729) & -139.5535) & -131.0121) & & 153.1876) \\ (\infty, \infty) & (119.3972 & (76.4120 & (132.0297 & (\infty, \infty) & (87.4756 \\ & -96.4129) & -133.0602) & -116.0185) & & -174.3343) \\ (-155.0228 & (111.6907 & (70.1076 & (119.3622 & (54.7031 & (67.1420 \\ -56.0737) & -88.7063) & -126.7558) & -103.3509) & 155.9307) & -154.0007) \\ (179.6319 & (104.1675 & (63.9049 & (107.7237 & (28.9060 & (49.3186 \\ -30.7283) & -81.1832) & -120.5531) & -91.7124) & -178.2722) & -136.1773) \\ (157.2088 & (96.6906 & (57.7283 & (96.4831 & (5.8412 & (31.7681 \\ -8.3052) & -73.7063) & -114.3765) & -80.4718) & -155.2074) & -118.6267) \\ (133.5205 & (89.1302 & (51.5052 & (85.1826 & (-19.0310 & (12.5894 \\ 15.3831) & -66.1458) & -108.1534) & -69.1713) & -130.3352) & -99.4481) \\ (100.0374 & (81.3428 & (45.1578 & (73.3407 & (-59.2204 & (-13.0112 \\ 48.8662) & -58.3584) & -101.8060) & -57.3294) & -90.1458) & -73.8475) \\ (\infty, \infty) & (73.1425 & (38.5935 & (60.2219 & (\infty, \infty) & (\infty, \infty) \\ & -50.1582) & -95.2417) & -44.2107) & & \\ (\infty, \infty) & (64.2466 & (31.6897 & (44.0995 & (\infty, \infty) & (\infty, \infty) \\ & -41.2622) & -88.3379) & -28.0882) & & \\ (\infty, \infty) & (54.1330 & (24.2649 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & -31.1486) & -80.9131) & & & \\ (\infty, \infty) & (41.4954 & (16.0109 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & -18.5111) & -72.6591) & & & \\ (\infty, \infty) & (16.3694 & (6.2894 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & 6.6149) & -62.9376) & & & \\ (\infty, \infty) & (\infty, \infty) & (-6.8745 & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ & & -49.7737) & & & \\ (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \\ (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) & (\infty, \infty) \end{pmatrix}$$

TABLE II

Membership Association Matrix In Determining Integer Ambiguity

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

NovAtel GPS receivers are selected for GPS data measurement. The GPS measurements are transmitted to the PC through a coaxial cable. The GPS carrier phase and navigation data are then recorded and processed.

Several tests are conducted. In the following, a test case, Case 3, is discussed. There are seven GPS satellites being simultaneously observed by the two GPS receivers in this case. Therefore, there are six double differences as shown in Fig. 4. Using the ambiguity function method to process the ensemble of data, an azimuth angle ψ is resolved to be 206.3023° or -153.6977° . Fig. 5 depicts the attitude determination using the ambiguity function method approach for this case.

The neural network approach is also conducted to determine the attitude. Considering the L_1 carrier frequency and a baseline length of 1 m, we have

$$\sqrt{(P'(t_k))^2 + (Q'(t_k))^2} \leq 10.526.$$

Hence, the integer N_{mn}^{ij} ranges between -10 and 11 . In Case 3, the matrix H that carries information concerning candidate attitudes can then be constructed and is tabulated in Table I. After the training of the neural network, the configuration or membership association matrix V is determined as shown in Table II. The integer as well as the azimuth can then be read out. The azimuth angle is found to be -153.9670° . Note that the neural network method only processes the measurement at one epoch rather than the whole ensemble of data as the ambiguity function method does. The two methods give consistent results.

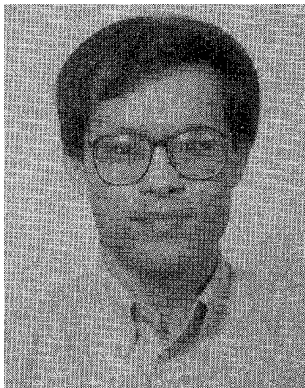
V. CONCLUSIONS

In this paper, the governing equation that characterizes GPS carrier phase double difference measurements, integer ambiguities, and attitude is developed. Two attitude determination algorithms are then proposed to solve the attitude and integer ambiguity. One algorithm explores the ambiguity function method and converts the problem as a parameter optimization problem. Another algorithm relies on the CHNN to evolve to a stable configuration which provides the integer ambiguity and attitude information. An experiment is conducted to verify the two algorithms. Satisfactory results have been obtained. Future work is planned to extend the algorithms to three axes attitude determination problem.

REFERENCES

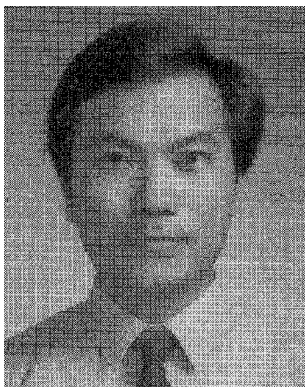
- [1] Siouris, G. M. (1993) *Aerospace Avionics Systems: A Modern Synthesis*. New York: Academic Press, 1993.
- [2] Wertz, J. R. (Ed.) (1978) *Spacecraft Attitude Determination and Control*. Boston: Kluwer Academic Publishers, 1978.
- [3] Grass, F. V., and Brash, M. (1992) GPS interferometric attitude and heading determination: Initial flight test results. *Navigation: Journal of the ION*, **38**, 4 (1991–1992).
- [4] Neira, M. M., and Lucas, R. (1992) GPS attitude determination of spin stabilized satellites. *Proceedings of the Institute of Navigation GPS-92*, Sept. 1992.
- [5] Nesbo, I., and Canter, P. (1990) GPS attitude determination for navigation. *GPS World*, 1990.
- [6] Lucas, R., Okkes, R., and Kriedte, W. (1988) Attitude determination with GPS. In *Proceedings of the Satellite Division's International Technical Meeting*, Colorado Springs, CO, 1988, 85–88.
- [7] Brown, R. A. (1992) Instantaneous GPS attitude determination. In *IEEE Aerospace and Electronics Magazine*, **7**, 6 (June 1992), 3–8.
- [8] Quinn, P. G. (1993) Instantaneous GPS attitude determination. *Proceedings of the Institute of Navigation GPS-93*, 1993, 603–615.
- [9] Diefes, D. L., Hazel, G. G., and Greenlee, D. F. (1993) GPS attitude determining system for marine navigation. *Proceedings of the Institute of Navigation GPS-93*, 1993, 649–655.
- [10] Hwang, P. Y. C. (1990) A near-instantaneous initialization method for attitude determination. *Proceedings of the Institute of Navigation GPS-90*, Colorado Spring, CO, 1990, 661–669.
- [11] Hatch, R. (1989) Ambiguity resolution in the fast lane. In *Proceedings of the Institute of Navigation GPS-89*, 1989, 45–50.

- [12] Chen, D. (1993)
Fast ambiguity search filter: A novel concept for GPS ambiguity resolution.
Proceedings of the Institute of Navigation GPS-93, 1993, 781–787.
- [13] Axelrad, P., Chesley, B. C., and Ward, L. M. (1993)
GPS based spacecraft attitude determination.
Final Report for October 1992–September 1993 to the Naval Research Laboratory. University of Colorado, Boulder, Sept. 1993.
- [14] Hofmann-Wellenhof, B., Lichtenegger, H., and Collins, J. (1993)
GPS Theory and Practice.
New York: Springer-Verlag, 1993.
- [15] Cichocki, A., and Unbehauen, R. (1993)
Neural Networks for Optimization and Signal Processing.
New York: Wiley, 1993.
- [16] Lemmon, M., and Vijaya Kumar, B. V. K. (1992)
Competitive learning with generalized winner-take-all activation.
IEEE Transactions on Neural Networks, **3**, 2 (Mar. 1992), 167–175.



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